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AUTHOR	Thompson, Russ; Fuller, Albert
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ABSTRACT

This teacher guide is part of the materials prepared for an individualized program for ninth-grade algebra and basic mathematics students. Materials written for the program are to be used with audiovisual lessons recorded on tape cassettes. For an evaluation of the program see ED 086 545. In this guide, the teacher is provided with objectives for each topic area and guided to materials written for a given topic. Three short criterion tests are included for each topic covered. Techniques are presented in this package for solving equations and inequalities constructed with algebraic fractions. This work was prepared under an ESEA Title III contract. (JP)

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ED 090 020

ALGEBRA I

Package #03-09

USING FRACTIONS

Prepared By

Russ Thompson and Albert Fuller

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SE 017 573

Many problems, when translated into mathematical symbols, result in equations or inequalities that have fractions in them. In this package we will use information learned in previous packages to solve problems of this type. You should be especially familiar with the transformations previously used to solve equations and inequalities (Packages 4, 5) and also the package on fundamental operations with fractions. (Package 8)

You may have heard people say that they never have used algebra to solve problems sinch they left school. The chances are that such people either do not have any mathematical problems to solve, or they never learned how to solve problems algebraically.

In order for algebra to be useful you must learn how to solve problems algebraically.

The goal of this package:

Given a verbal problem leading to a fractional equation or inequality, translate it into mathematical symbols and solve the resulting statement.

PACKAGE OBJECTIVES:

1. Given an equation or an inequality in which fractions occur, solve it.
2. Given a per cent mixture problem, solve it.
3. Given a problem involving use of the simple interest formula, solve it.
4. Given a fractional equation, solve it.
5. Given a rate-of-work type problem, solve it.
6. Given a rate-of-motion problem, solve it.

I. U. #03-09-01

SOLVING EQUATIONS AND INEQUALITIES

You will need to recall:

That when each member of an inequality is multiplied or divided by a negative number, the direction of the inequality is reversed.

OBJECTIVES:

1. Given an equation or an inequality in which fractions occur, solve it.

ACTIVITIES:

1. Study page 329 in SM. Use the example solution number one. If you wish to try solution number 2 the explanation is in error. IN solution number 2 just add the fractions as usual and solve the resulting simplified equation by means of transformation as in Package 4.
(Objective 1)

2. Do the odd numbered part A exercises on page 330, SM. You should be able to do some of the part B and C exercises too.
(Objective 1)

Criterion Test 03-09-01-01

1. Solve.

(A) $\frac{a}{3} + \frac{a}{6} = 1$

(B) $\frac{4x}{5} - \frac{3x}{10} > \frac{3}{2}$

(C) $\frac{1}{3^x} + \frac{1}{2^x} = \frac{1}{6^x} + 1$

(D) $\frac{x}{4} \leq \frac{2x}{5} + 1$

(E) $\frac{y^2}{3} - \frac{17y}{9} - \frac{2}{3} = 0$

(F) $a^2 - \frac{13a}{21} + \frac{2}{21} = 0$

Criterion Test 03-09-01-02

1. Solve.

(A) $\frac{x}{4} + \frac{x}{8} = 1$

(B) $\frac{5x}{4} - \frac{3x}{8} > \frac{3}{2}$

(C) $\frac{1}{2}x - \frac{1}{4}x = \frac{1}{8}x + 1$

(D) $\frac{x}{3} \leq \frac{2x}{5} + 1$

(E) $\frac{x^2}{8} + \frac{x}{8} - \frac{3}{2} = 0$

(F) $\frac{x^2}{5} - \frac{12x}{15} - 1 = 0$

Criterion Test 03-09-01-03

1. Solve.

(A) $\frac{a}{2} + \frac{a}{4} = 1$

(B) $\frac{3b}{4} - \frac{3b}{2} < \frac{3}{2}$

(C) $\frac{1}{5}c + \frac{1}{3}c = \frac{1}{15}c + 1$

(D) $\frac{d}{5} \leq \frac{3d}{10} + 1$

(E) $\frac{x}{3} + \frac{2x}{4} - \frac{3}{2} = 0$

(F) $\frac{x^2}{2} - \frac{9x}{4} + 1 = 0$

Answers to Criterion Tests

Test 03-09-01-01

1. (A) $a = 2$ (B) $x > 3$ (C) $x = 1\frac{1}{2}$

   (D) $x \geq -6\frac{2}{3}$ (E) $y = -\frac{1}{3}$, $y = 6$

   (F) $a = \frac{1}{3}$, $a = \frac{2}{7}$

Test 03-09-01-02

1. (A) $x = 2\frac{2}{3}$ (B) $x > \frac{12}{7}$ (C) $x = 8$

   (D) $x \geq -15$ (E) $x = -4$, $x = 3$

   (F) $x = -1$, $x = 5$

Test 03-09-01-03

1. (A) $a = \frac{4}{3}$ (B) $b > -2$ (C) $c = \frac{15}{7}$ or $2\frac{1}{7}$

   (D) $d \geq -10$ (E) $x = \frac{9}{5}$

   (F) $x = \frac{1}{2}$, $x = 4$

I. U. #03-09-02

PERCENT MIXTURE PROBLEMS

OBJECTIVES:

1. Given a percent mixture problem, solve it.

ACTIVITIES:

1. Study page 331 s + m, and work the odd numbered Part A problems on page 332. Try a few of the part B and C problems on page 333 to be sure that you have met the objective.

Criterion Test 03-09-02-01

1. Solve the following problems.

   (A) How many ounces of a 75% acid solution must be added to 60 ounces of a 15% acid solution to produce a 50% acid solution?

   (B) How many pounds of water must be added to ten pounds of a 10% salt solution in order to produce a 6% salt solution?

   (C) How many pounds of water must be evaporated from 80 pounds of a 4% salt solution in order to obtain a 6% solution?

Criterion Test 03-09-02-02

1. Solve the following problems.

   (A) How many ounces of water must be added to six ounces of a 50% antiseptic solution to produce a 40% solution?

   (B) How many ounces of water must be added to 2 ounces of a 60% antifreeze solution to produce a 20% solution?

   (C) How many pounds of water must be evaporated from 60 pounds of brine to change a 6% brine solution to a 10% brine solution?

Criterion Test 03-09-02-03

1. Solve the following problems.

   (A) How many pounds of pure alcohol must be added to 10 pounds of an 80% pure alcohol solution to produce an 85% pure alcohol?

   (B) How many ounces of water must be added to two ounces of a 30% brine solution to produce a 20% solution?

   (C) How many ounces of a 60% acid solution must be added to 5 ounces of a 10% acid solution in order to produce a 50% solution?

Answers to Criterion Tests

Test 03-09-02-01

1. (A) $x = 84$ ounces (B) $x = 6.\overline{6}$ pounds of water

(C) $x = 26.\overline{6}$ pounds

Test 03-09-02-02

1. (A) $x = 1\frac{1}{2}$ ounces to be added

(B) $x = 4$ ounces water to be added

(C) $x = 24$ pounds of water

Test 03-09-02-03

1. (A) $x = 3\frac{1}{3}$ pounds

(B) $x = 1$ ounce

(C) $x = 20$ ounces

I. U. #03-09-03

INVESTMENT PROBLEMS

OBJECTIVES:

1. When asked to write the simple interest formula, write "i = prt".

2. Given a problem involving use of the simple interest formula, solve it.

ACTIVITIES:

1 1. Study page 333 and 334 in S + M, and do the odd numbered part A exercises on pages 334, 335.

2. Try some of the part B and C problems. If you like money you'll find them especially interesting.

Criterion Test 03-09-03-01

1. Write the simple interest formula.

2. Solve the following problems.

   (A) If $2500.00 is invested at 7% how much simple interest is earned in 3 years?

   (B) Jack has some money invested at 8% interest. If he adds $2000 to his investment his annual interest will amount to $880.00. How much did Jack have invested before he added the $2000.00?

   (C) If a total of $8000.00 is invested, part at 6% and part at 8%, and the annual income is $590.00, find the amount invested at each rate.

Criterion Test 03-09-03-02

1. Write the simple interest formula.

2. Solve the following problems.

   (A) If I invest $750.00 at 6% simple interest, how much will the income for three years be?

   (B) An investment at 6% simple interest returned 58.50 in three years. How much was the original investment.

   (C) $9000.00 was invested part at 4% simple interest and part at 6% simple interest. If the annual imcome was $520.00, how much was invested at 4%?

Criterion Test 03-09-03-03

1. Write the simple interest formula.

2. Solve the following problems.

   (A) If $800.00 is borrowed at 8% simple interest, how much money must be paid back after five years?

   (B) Joe had $8300.00 in a savings account at 4% simple interest. After withdrawing some of the money to pay for a hi-fi stereo his annual income from his savings was $320.00 per year. How much did he withdraw?

   (C) Mike McKinney invested $5000.00 part at 3% and part at 5% simple interest. His annual income from these investments is $200.00. How much does he have invested at each rate?

Answers to Criterion Tests

Test 03-09-03-01

1. i = prt

2. (A) $325.00 (B) $9000.00

   (C) $2500.00, $5500.00

Test 03-09-03-02

1. i = prt

2. (A) $135.00 (B) $325.00 (C) $1000.00

Test 03-09-03-03

1. i = prt

2. (A) $1120 (B) $300.00 (C) $2500.00 at each rate

I. U. #03-09-04

SOLVING FRACTIONAL EQUATIONS

OBJECTIVES:

1. When asked to define a fractional equation write, "A fractional equation is an equation which has a variable in the denominator of one or more terms".

2. When asked to quote the caution to be observed in solving fractional equations, write, "Only values producing true statements when substituted in the original equation belong to the solution set".

3. Given a fractional equation, solve it.

ACTIVITIES:

1. Study page 336, S + M.
   (Objectives 1, 3)

2. Study page 337 and write the odd numbered part A exercises on page 337.
   (Objectives 2, 3)

Criterion Test 03-09-04-01

1. Define a fractional equation.

2. What caution should be observed when solving fractional equations?

3. Solve the following fractional equations.

   (A) $\frac{3}{x} - 2 = \frac{5}{2x} - \frac{3}{2}$ (B) $\frac{1}{x^2 - x} = \frac{3}{x} - 1$

Criterion Test 03-09-04-02

1. Define a fractional equation.

2. What caution should be observed when solving fractional equations.

3. Solve the following fractional equations.

   (A) $\frac{x - 2}{5x} = \frac{1}{5} - \frac{4}{15x}$ (B) $\frac{x - 5}{8x} = \frac{3}{x + 5}$

Criterion Test 03-09-04-03

1. Define a fractional equation.

2. What caution should be observed when solving fractional equations?

3. Solve the following fractional equations.

   (A) $\frac{2}{3x} = \frac{2}{x + 4}$ (B) $\frac{4}{3x} + \frac{3}{3x + 1} = -2$

Answers to Criterion Tests

Test 03-09-04-01

1. A fractional equation is an equation which has a variable in the denominator of one or more terms.

2. Only values producing true statements when substituted in the original equation belong to the solution set.

3. (A) $x = 1$ {1} (B) $x = 2$ {2}

Test 03-09-04-02

1. A fractional equation is an equation which has a variable in the denominator of one or more terms.

2. Only values producing true statements when substituted in the original equation belong to the solution set.

3. (A) $x = 4$ {4} (B) $x = 25$ or $x = -1$ {25, -1}

Test 03-09-04-03

1. A fractional equation is an equation which has a variable in the denominator of one or more terms.

2. Only values producing true statements when substituted in the original equation belong to the solution set.

3. (A) $x = 2$ {2} (B) $x = -\frac{1}{6}$ or $x = -\frac{4}{3}$

$\left\{-\frac{1}{6}, -\frac{4}{3}\right\}$

I. U. #03-09-05

RATE-OF-WORK PROBLEMS

OBJECTIVES:

1. When asked to write the formula used in rate-of-work problems write, "w = rt"

2. Given a rate-of-work problem, solve it.

ACTIVITIES:

1. Study pages 338 and 339 in S + M, and do the odd numbered part A problems on page 339. You should also be able to do some of the more challenging part B and C problems.
(Objectives 1, 2)

Criterion Test 03-09-05-01

1. Write the formula used in rate-of-work problems.

2. Solve the following problems.

   (a) A battery can operate one flashlight four times as long as it can operate another flashlight. If both flashlights are wired to the battery at the same time, the battery will last 16 hours. How long would the battery operate each flashlight alone?

   (b) One pipe can fill a 24 foot cattle tank in 12 hours while another can fill it in six hours. How long would it take for both pipes together?

   (c) One man, working alone, can build ½ mile of fence in forty-eight hours; his son could do it alone in twice that time. How long will it take them working together?

Criterion Test 03-09-05-02

1. Write the formula used in rate-of-work problems.

2. Solve the following problems.

   (a) An electric hot water heater with one of its heating elements in operation can raise the temperature of 80 gallons of water fifteen degrees in twenty-four minutes. With its second heating element in operation also, it can raise the temperature of 80 gallons of water 15 degrees in six minutes. If the second heating element were operating alone, how long would it take to raise the temperature of the 80 gallons of water 15°?

   (b) One of the floodgates at Johnson lake can lower the water level ten feet in twenty-four hours. A second floodgate can lower the level ten feet in thirty hours. How long would it take to lower the water level ten feet if both floodgates were opened?

   (c) The Arnold water tower is filled from two separate wells with two different electric pumps. Pump A can fill the reservoir in eight hours. The older pump (pump B) can fill it in twelve hours. How long will it take both pumps working together?

Criterion Test 03-09-05-03

1. Write the rate-of-work formula.

2. Solve the following problems.

   (a) Tractor A can disc a field in 14 hours. Tractor B can disc it in eight hours. If both tractors disc the field at the same time, how long will it take to go over it once?

   (b) One student can do a set of math problems in ½ the time it takes another student to do the same set of problems. If it takes them 20 minutes when working together, how long would it take each student working alone?

   (c) Ty Pryter can type math problems at an average rate of twenty problems per hour. Johny Fumble Fingers can type twenty problems in four hours. How long will it take Pryter and Fingers to type twenty problems when working together?

Answers to Criterion Tests

Test 03-09-05-01

1. $w = rt$

2. (a) $x = 20$, $4x = 80$ {20 hours, 80 hours}

   (b) $x = 4$ {4 hours}

   (c) $x = 32$ {32 hours}

Test 03-09-05-02

1. $w = rt$

2. (a) $x = 8$ {8 minutes}

   (b) $x = 13.3$ {$13\frac{1}{3}$ hours}

   (c) $x = 4\frac{4}{5}$ {4 hours and 48 minutes}

Test 03-09-05-03

1. $w = rt$

2. (a) $x = 5\frac{1}{11}$ {$5\frac{1}{11}$ hours}

   (b) $x = 30$, $2x = 60$ {30 minutes, 60 minutes}

   (c) $x = \frac{4}{5}$ {$\frac{4}{5}$ hour or 48 minutes}

I. U. #03-09-06

MOTION PROBLEMS

OBJECTIVES:

1. Given a rate-of-motion problem, solve it.

ACTIVITIES:

1. Review pages 188 and 189 in S + M. (Background for Objective 1)

2. Study pages 340 and 341, S + M and work the odd numbered part A problems. If you like challenges try the B and C problems.
(Objective 1)

Criterion Test 03-09-06-01

1. Solve the following problems:

   (a) I can row three miles per hour in still water. If it takes me twice as long to row 8 miles upstream as it does to row 8 miles down stream, what is the rate of flow of the current?

   (b) Water flows down the Platte river at 2 miles per hour. I can row 12 miles downstream in the same time it takes me to row 4 miles upstream. How fast am I rowing?

   (c) An airplane whose cruising speed is 220 miles per hour in still air can fly from Omaha to Podunk, a distance of 520 miles, due east, in the same length of time it would take to fly to Tombstone, a distance of 360 miles due west. What is the speed of the wind if it is blowing from west to east?

Criterion Test 03-09-06-02

1. Solve the following problems.

   (a) I can paddle my canoe six miles per hour in still water. If it takes four times as long to paddle nine miles upstream as it takes to paddle 9 miles down stream, how fast is the river flowing?

   (b) The Niobrara river runs at an average rate of nine miles per hour south of Valentine, Nebraska. I can paddle nine miles down stream in the same time that it takes to paddle one mile upstream. How fast can I paddle in still water?

   (c) The Red Barron flies his aircraft 200 miles east and back again. If he can fly 150 miles per hour in still air and it takes him twice as long to fly back, how fast is the wind blowing?

Criterion Test 03-09-06-03

1. Solve the following problems.

   (a) The Brownville Belle, a river boat that operates on the Missouri river, travels at a speed of 12 m.p.h. in still water. It can go 20 miles upstream in the same time it can go 60 miles downstream. How fast is the current?

   (b) A canoe floats down the South Loup river a distance of three miles in one hour. Joan can paddle three miles down stream in the same length of time it takes her to paddle half a mile upstream. How fast is she paddling?

   (c) Jim drove his car 300 miles in the same time that Ed drove his car 270 miles. Jim drove 5 miles per hour faster than Ed. Find the speed of each.

Answers to Criterion Tests

Test 03-09-06-01

1. (a) {1} 1 mile per hour

   (b) {4} 4 miles per hour

   (c) {40} 40 miles per hour.

Test 03-09-06-02

1. (a) {3.6} 3.6 miles per hour

   (b) {11¼} 11¼ miles per hour

   (c) {46 $\frac{5}{6}$} 46 $\frac{5}{6}$ miles per hour

Test 03-09-06-03

1. (a) {8.75} 8.75 miles per hour

   (b) {4 $\frac{1}{5}$} 4 $\frac{1}{5}$ miles per hour

   (c) {45, 50} Ed - 45 miles per hour
   Jim - 50 miles per hour

THE END
of
PACKAGE
03-09